

# 1 Partial Fractions

## 1.1 Concepts

1. Partial fractions allow us to compute an antiderivative of an expression of the form  $P(x)/Q(x)$ , where  $P, Q$  are polynomials, more easily (these are just fractions where the numerator and denominator are both polynomials). First long divide so that the degree or highest term of the polynomial  $P$  is less than  $Q$ . Then factor  $Q(x)$  into linear factors if you can, or else quadratic factors. Then for each factor, write the simplification of the

form:

Factor	$ax + b$	$(ax + b)^n$	$ax^2 + bx + c$	$(ax^2 + bx + c)^n$
Expression	$\frac{A}{ax+b}$	$\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \dots$	$\frac{Ax+B}{ax^2+bx+c}$	$\frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \dots$

Afterwards, find what these constants are. One good way to do this is to multiply everything by  $Q(x)$  to clear denominators and then plug in different values of  $x$ .

## 1.2 Problems

2. True    False    To find the partial fraction decomposition of  $\frac{4x^3}{(x-1)(x+2)^2}$ , we set it equal to  $\frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$  and solve for  $A, B, C$ .
3. Find  $\int \frac{x^2}{x^2 + 3x - 18} dx$ .
4. Find  $\int \frac{x^3 + 3x^2 + 3x + 3}{(x + 1)^2(x^2 + 1)} dx$ .
5. Integrate  $\int \frac{5x}{x^2 - 9x - 36} dx$ .
6. Integrate  $\int \frac{4x^2}{(x - 1)(x - 2)^2} dx$ .
7. Integrate  $\int \frac{3x^2 - x}{(x - 1)(x^2 + 1)}$ .
8. Set up the partial fraction decomposition of  $\frac{8x^3 + 3x^2 + 1}{(x - 1)^2(x^2 + 4)^2}$  (you don't have to solve for the coefficients).
9. Integrate  $\int \frac{\sec^2(x)}{\tan(x)^2 - \tan(x)} dx$ .